A METHOD TO CALCULATE AN ELECTRIC-ARC GENERATOR WITH VORTEX GAS STABILIZATION OF THE ARC

B. D. Voronin, M. Ya. Smelyanskii, and A. M. Tsirlin

Inzhenerno-Fizicheskii Zhurnal, Vol. 13, No. 5, pp. 656–662, 1967

UDC 537.525.1:537.523.5

Methods are presented for generalization of the volt-ampere characteristics and calculation of working parameters for a hydrogen electric-arc heater with vortex arc stabilization.

Substantial experimental material has been accumulated to the present time in connection with the study and application of electric-arc generators with vortex gas stabilization of the arc, on the basis of which attempts are made to develop a method for calculating and simulating equipment of analogous design.

To derive generalized empirical relationships and to use these on a converted scale, the methods of the theory of similarity and dimensional analysis have been successfully employed [1–3].

In connection with generators of the design in question, the equation for the generalized volt-ampere characteristic in criterial form can be written as follows:

$$\frac{U\sigma_0 d}{I} = f\left(\frac{I^2}{Gh_0\sigma_0 d};\ \mathrm{Re};\ \Gamma\right), \qquad (1)$$

where Γ is a determining geometric parameter (or parameters).

It was noted earlier [4, 5], that in our experiments in the study of a hydrogen heater (see Fig. 1) the experimental data are satisfactorily generalized by an equation of the form

$$\frac{Ud}{I} = f\left(\frac{I^2}{Gd};\ \frac{I^{2/3}}{d};\ \mathrm{Re}\right). \qquad (2)$$

The dimensional simplex $I^{2/3}/d$, corresponding to the nondimensional criterion $\Gamma = d_{\mathrm{arc}}/d$, expresses the relationship between two determining geometric dimensions of the system. It can be derived from the determining criterion $\Pi_1 = \rho_0 W_0 h_0 / j_0 E_0 d$ [1], with consideration of the heat-transfer conditions at the boundary between the column of the arc and the gas vortex.

Bearing in mind that $w_0 = 4G/\pi d^2 \rho_0$ and $j_0 = 4I/\pi d_{\mathrm{arc}}^2$, we can write

$$\Pi_1 = \frac{\rho_0 w_0 h_0}{j_0 E_0 d} \sim \frac{\rho_0 G h_0 \sigma_0 d_{\mathrm{arc}}^4}{\rho_0 I d^3} \sim \frac{G h_0 \sigma_0 d}{I^2}\ \frac{d_{\mathrm{arc}}^4}{d^4} \sim \Pi_1' \Pi_1''. \qquad (3)$$

To determine d_{arc}, we use the heat-transfer equation which, for a unit length of column arc, is given in the following form:

$$Q = A_1 I^2 R_{\mathrm{arc}\,1} = \alpha F \Delta t = \alpha\pi d_{\mathrm{arc}} \Delta t, \qquad (4)$$

where $R_{\mathrm{arc}1} = C_1/d_{\mathrm{arc}}^2$; α is the reference heat-transfer coefficient.

In the general case, the quantities α and Δt are functions of the physical properties of the medium and of the heat-transfer conditions; for example, for the given gas $\alpha(\Delta t) = f(\mathrm{Re})$. Replacing $R_{\mathrm{arc}1}$, we can write

$$I^2/d_{\mathrm{arc}}^2 \sim C_2 d_{\mathrm{arc}} f(\mathrm{Re}) \ \text{ or } \ d_{\mathrm{arc}} \sim C_3 I^{2/3} f(\mathrm{Re}). \qquad (5)$$

Consequently, the geometric criterion d_{arc}/d assumes the form

$$\Pi_1'' \sim \frac{I^{2/3}}{d} f(\mathrm{Re}). \qquad (5a)$$

With insignificant variations in the Re number, as was the case in our experiments, we have

$$\Pi_1'' \sim I^{2/3}/d. \qquad (5b)$$

Since for a hydrogen heater with a sleeve cathode (d_c = 35 mm; d = 10–30 mm) the experimental data are satisfactorily generalized by the equation [5]

$$\frac{Ud}{I} = 4500\left(\frac{I^2}{Gd}\right)^{-0.33}\left(\frac{I^{2/3}}{d}\right)^{-1.0}, \qquad (6)$$

which can be expressed (for use in the calculations) in simpler form

$$U = 4500\left(\frac{I}{Gd}\right)^{-0.33}. \qquad (6a)$$

For a hydrogen heater with a pin cathode (from the more precise data) we derive the following empirical equation:

$$\frac{Ud}{I} = 3300\left(\frac{I^2}{Gd}\right)^{-0.29}\left(\frac{I^{2/3}}{d}\right)^{-1.0}. \qquad (7)$$

The experimental data can also be processed in the form of the function [6]

$$\frac{Ud}{I} = f\left(\frac{I^2}{Gd};\ \frac{G}{d};\ d\right), \qquad (8)$$

where G/d is a dimensional parameter corresponding to the Reynolds number, with the physical properties of the gas retained.

For comparison purposes Fig. 2 shows our experimental data for a generator with a pin cathode, processed according to Eqs. (7) and (8):

$$\frac{Ud}{I} = 2850\left(\frac{I^2}{Gd}\right)^{-0.6}\left(\frac{G}{d}\right)^{-0.3} d^{0.4}. \qquad (9)$$

As we can see from Fig. 2, in both of the cases satisfactory generalization has been achieved. Consequently, both equations reflect rather well the fundamental features of the subject generator. However, Eq. (7), and the more complete equation (2), are physically more valid.

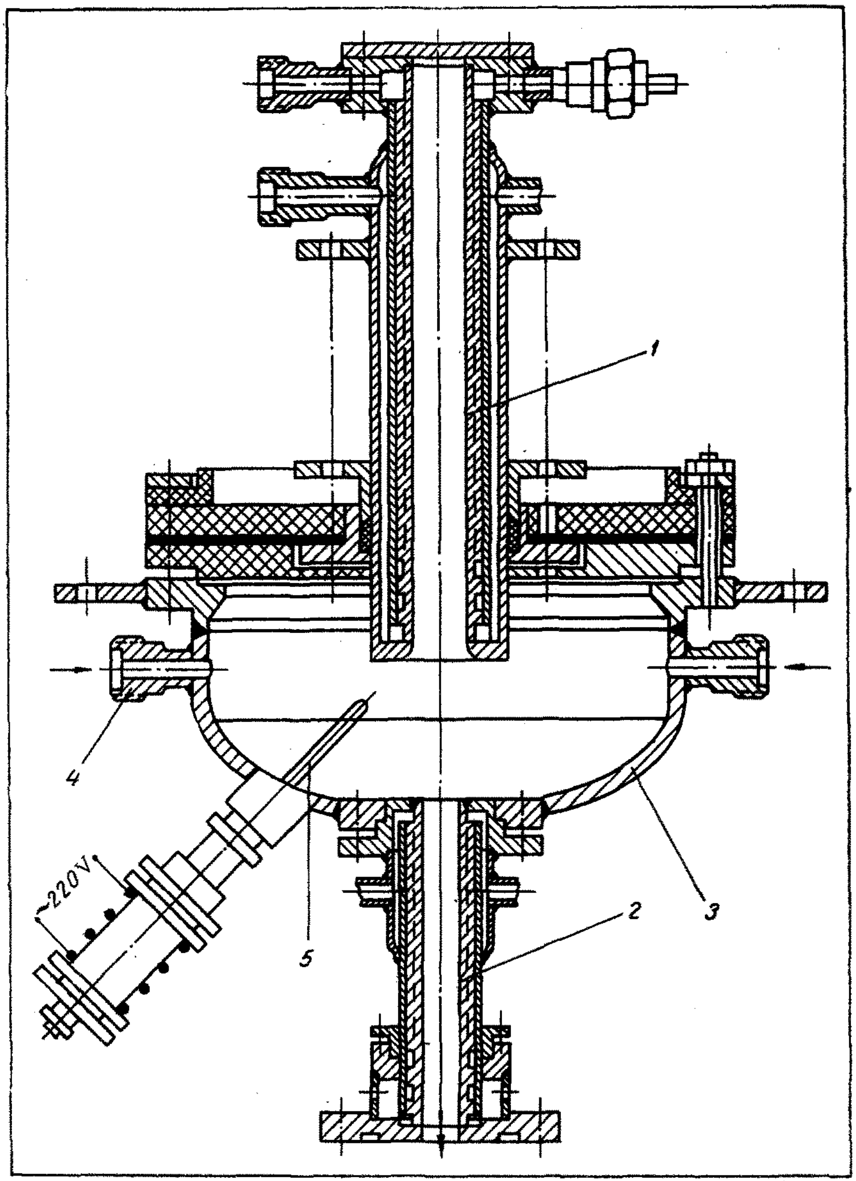

Fig. 1. Experimental model of a hydrogen electric arc heater. 1) Cathode; 2) anode; 3) vortex chamber; 4) tangential gas supply; 5) keep-alive electrode.

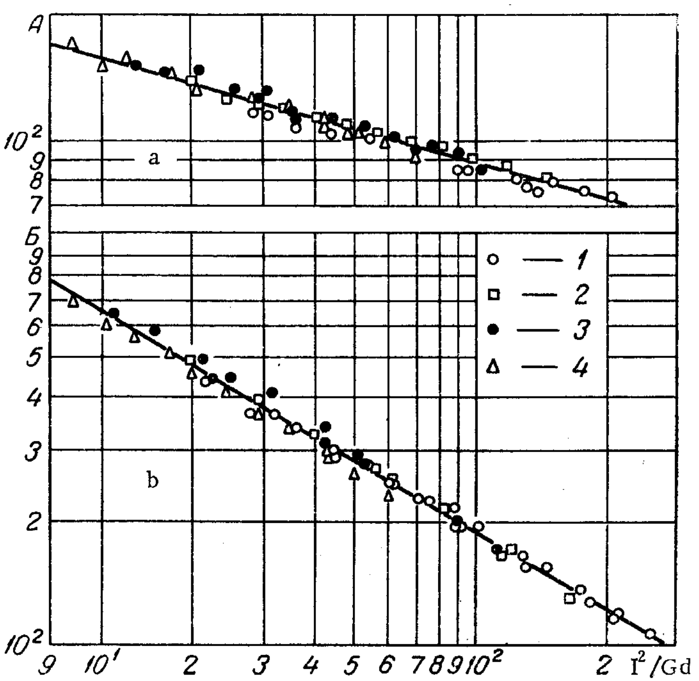

Fig. 2. Generalized volt-current characteristics of hydrogen heater with a pin cathode (a) according to Eq. (7); b) according to Eq. (9)); $A \equiv (Ud/I)(I^{2/3}/d)$; $B \equiv (Ud/I)(G/d)^{0.3}d^{-0.4}$:
1) d = 1.0 cm; 2) 1.5; 3) 2.0; 4) 3.0.

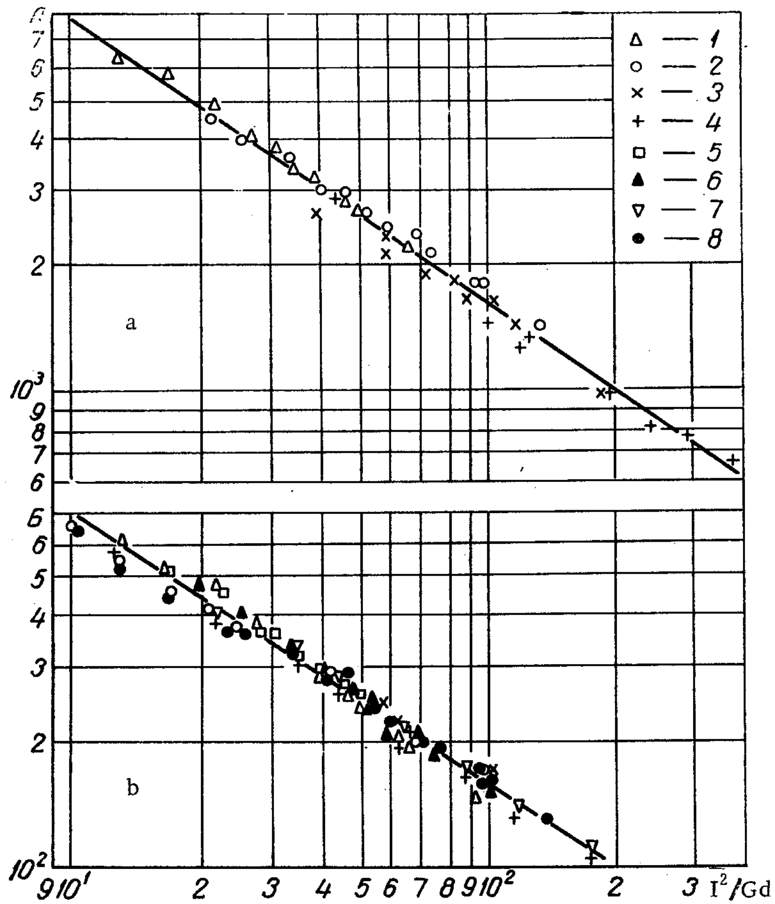

Fig. 3. Generalized volt-current characteristics of hydrogen heater with sleeve electrodes (a) according to Eq. (11); b) according to Eq. (10)); $A \equiv (Ud/I)(G/d)^{0.3}d^{-0.6}$; $B \equiv (Ud/I)(G/d)^{0.31}d^{-0.5}(d_c/d)^{-0.29}$):
1) d_c/d = 42/30 mm/mm; 2) 35/20; 3) 27/15; 4) 18/10; 5) 27/30; 6) 42/20; 7) 35/10; 8) 27/20.

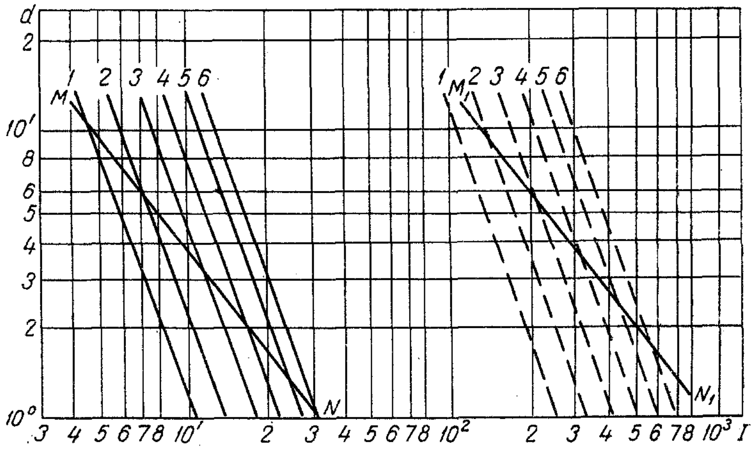

Fig. 4. Dependence of output electrode diameter d (cm) on current I (A) and specific power: 1) N_{spec} = 20 000 W · sec/g; 2) 25 000; 3) 30 000; 4) 35 000; 5) 40 000; 6) 45 000; MN and M_1N_1 are the limiting value boundary lines of electrode diameter for hydrogen rate G = 1 and 30 g/sec.

In generalizing the experimental results for a generator with two tubular electrodes, the criterial equation may be expanded with another geometric parameter expressing the relationship between the electrode diameters [6]. The effect of distance between the electrodes must be taken into consideration in analogous fashion, while for generators with a neutral insert, consideration must be given to the relative dimension of the latter.

However, if we assume in the simulation that the fundamental geometric relationships are preserved (for example, $d_c/d = 1.5$, $l_{a-c}/d = 2$), the theoretical [computational] equations will include only the determining geometric parameter (d_{arc}/d or d).

Figure 3 shows the experimental data derived in the study of a hydrogen heater with a sleeve cathode (d_c = = 18–42 mm). With consideration of the geometric parameter d_c/d (l_{a-c} = 60 mm) the experimental points are approximated by the equation

$$\frac{Ud}{I} = 3250 \left(\frac{I^2}{Gd}\right)^{-0.66} \left(\frac{G}{d}\right)^{-0.31} d^{0.5} \left(\frac{d_c}{d}\right)^{0.29}. \tag{10}$$

However, if $d_c/d = 1.4–1.6$ and $l_{a-c} = 2$, i.e., if they have values which are expediently applied in scale conversion [5], we derive the expression

$$\frac{Ud}{I} = 3450 \left(\frac{I^2}{Gd}\right)^{-0.68} \left(\frac{G}{d}\right)^{-0.3} d^{0.6}, \tag{11}$$

or on generalization according to formula (2), proposed by us,

$$\frac{Ud}{I} = 5450 \left(\frac{I^2}{Gd}\right)^{-0.4} \left(\frac{I^{2/3}}{d}\right)^{-0.9}. \tag{12}$$

To calculate the fundamental working parameters for a generator of the type in question, we have proposed [5] a system of three equations which, in addition to Eq. (1), includes the equations of gas dynamic arc stabilization and heater energy balance:

$$d = f\left(\frac{G}{I}\right), \tag{13}$$

$$N_{arc} = UI = \frac{N_{spec}\, G}{\eta}. \tag{14}$$

For the hydrogen heater with pin cathode that we have studied we obtained the following quantitative expression for Eq. (13), with a constant distance between the electrodes (l_{a-c} = 60 mm):

$$d \leqslant 61 \left(\frac{G}{I}\right)^{1.2}. \tag{15}$$

It should be noted that the numerical coefficients in Eq. (15) are not universal and must be adjusted for other geometric relationships.

Consequently, with regard to the given model of the apparatus, we assume Eq. (7) or Eqs. (9), (14), and (15) as the theoretical equations whose solution will make it possible to determine the values of U, I, and d.

Analysis of the system of theoretical equations makes it possible to derive expressions for the current strength and for the diameter of the output electrode as a function of the thermal load, i.e., of the gas flow rate and of the specific power (the temperature increment), which are generally given as the initial parameters.

Simultaneous solution of Eqs. (7), (14), and (15) for I and d for the specific case investigated by us yields

$$I_{theor} = \left(\frac{N_{spec}}{3300\, \eta}\right)^{1.31} \frac{G^{0.93}}{d^{0.37}}, \tag{16}$$

$$d_{cr} = \left(\frac{45000\, \eta}{N_{spec}}\right)^{2.85} G^{0.14}. \tag{17}$$

The thermal efficiency in the theoretical equations may be assumed within comparatively narrow limits (η = 0.7–0.9), which is characteristic of low-current generators ($I^{2/3}/d \leq 10$).

The functions expressed by Eqs. (16) and (17) are shown in graphical form in Fig. 4. The straight lines 1–6 are constructed on the basis of (16) for the hydrogen flow rate G = 1 g/sec and for a specific power of 20 000 to 45 000 W · sec/g. The straight line MN is constructed on the basis of (17) and corresponds to the limit values of the output-electrode diameter. Consequently, the straight line MN represents the boundary line between the region of stabilization lying to the left, and the region of unstable generator operatoring regimes.

Analysis of the relationships in the nomogram shows that for substantial thermal loads, when it is necessary to achieve a high temperature for the gas flow, it is necessary to employ output-electrode diameters close to their limit values (the straight line MN).

Conversely, for low and medium values of the specific power, the maximum theoretical electrode diameter may be exceedingly large ($d_{cr} > 100$ mm). Its size will then be chosen on the basis of structural and engineering considerations (smaller than d_{cr}), with the current strength determined on the basis of the diameter value selected from the graph or according to formula (16).

The dashed lines refer to a hydrogen flow rate of 30 g/sec and are constructed on the basis of the above-cited theoretical equations for the same values of the specific power as for the experimental apparatus.

It may be assumed that in providing for similar conditions of arc burning (the properties of the medium, the current density ($I^{2/3}/d$), the gas dynamic regime, geometric relationships), we will apparently have a rather valid basis for the possibility of extrapolation on scale conversion. Moreover, the validity of this extrapolation, as well as the values of the coefficients in the theoretical equations, will be refined in the testing of the hydrogen electric-arc heater with a capacity of 30 g/sec and a power of 1000 kW.

NOTATION

U is the voltage in V; I is the current in A; G is the gas flow rate, g/sec; σ_0, E_0, j_0, h_0, ρ_0, and w_0 are the determining values of electrical conductivity, electrical field intensity, current density, enthalpy, density, and gas velocity, respectively; d_{arc} is the arc diameter; d is the diameter of the output electrode-anode; d_c is the diameter of a sleeve cathode; l_{a-c} is the distance between electrodes; N_{spec} is the specific power, W sec/g; R_{arc1} is the electric resistance of unit arc column; Re is the Reynolds number; η is the thermal efficiency.

REFERENCES

1. O. I. Yas'ko, IFZh, no. 12, 1964.
2. G. Yu. Dautov and M. F. Zhukov, PMTF [Journal of Applied Mechanics and Technical Physics], no. 2, 1965.
3. G. Yu. Dautov, M. F. Zhukov, A. S. Koroteev, V. Ya. Smolyakov, Yu. I. Sukhinin, and O. I. Yas'ko, Transactions of the International Symposium on the Properties and Application of Low-Temperature Plasma [in Russian], Moscow, 1965.
4. B. D. Voronin, A. M. Tsirlin, and M. Ya. Smelyanskii, IFZh [Journal of Engineering Physics], **10**, no. 3, 1966.
5. B. D. Voronin, A. M. Tsirlin, and M. Ya. Smelyanskii, Khim. promyshlennost, no. 7, 1966.
6. A. I. Zhidovich, O. A. Savel'ev, and O. I. Yas'ko, IFZh [Journal of Engineering Physics], **11**, no. 5, 1966.

4 April 1967

Institute of the Chemistry
and Technology of Elementary Organic Compounds,
Moscow